LAMBLEY PRIMARY SCHOOL

Reaching for the Stars

Calculation Policy

Early Years Foundation Stage: F2

The principal focuses of mathematics in F2 are:

- Playing and exploring - children investigate and experience things, and ‘have a go’
- Active learning - children concentrate and keep on trying if they encounter difficulties, and enjoy achievements
- Creating and thinking critically - children have and develop their own ideas, make links between ideas, and develop strategies for doing things.

Each area of learning and development is implemented through planned, purposeful play and a mix of adult-led and child-initiated activity. Children are provided with a stimulating environment which supports their continuing development as confident mathematicians.

Key Stage 1

- The principal focus of mathematics teaching in key stage 1 is to ensure that pupils develop confidence and mental fluency with whole numbers, counting and place value. This should involve working with numerals, words and the four operations, including with practical resources.

Lower Key Stage 2 – years 3 and 4

- The principal focus of mathematics teaching in lower key stage 2 is to ensure that pupils become increasingly fluent with whole numbers and the four operations, including number facts and the concept of place value. This should ensure that pupils develop efficient written and mental methods and perform calculations accurately with increasingly large whole numbers.
- By the end of year 4, pupils should have memorised their multiplication tables up to and including the 12 multiplication table and show precision and fluency in their work.

Upper Key Stage 2 – years 5 and 6

- The principal focus of mathematics teaching in upper key stage 2 is to ensure that pupils extend their understanding of the number system and place value to include larger integers. This should develop the connections that pupils make between multiplication and division with fractions, decimals, percentages and ratio. Pupils should develop their ability to solve a wider range of problems, including increasingly complex properties of number and arithmetic, and problems demanding efficient written and mental methods and calculation. With this foundation, pupils are introduced to the language of algebra as a means for solving a variety of problems.
- By the end of year 6, pupils should be fluent in written methods for all four operation, including long multiplication and division, and in working with fractions, decimals and percentages.

Foundation Stage: F2
Addition

Key vocabulary:

Add, more, make, sum, total, altogether, double

How many more to make …?

How many more is… than…?

… is the same as…

Strategies:

- Combining two parts to make a whole: part whole model
- Starting at the bigger number and counting on

Guidance/ Models and Images

Numicon shapes are introduced and used to:

- Identify 1 more/ less
- Combine pieces to add
- Find number bonds
- Add without counting
- Children can record this by printing or drawing around Numicon

Children begin to combine groups of objects using concrete apparatus

Children are encouraged to place objects in a row as a link to the bar model

Construct number sentences verbally or using cards to match practical resources

Introduce = as *equal to, the same as, balanced to*

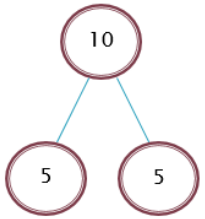
Children are encouraged to read number sentences in different ways:

"Three add two equals five" "Five is equal to three and two"

Solve simple problems using fingers

Use bead strings to support number bonds

Show addition on the part, part, whole model – "2 is a part, 3 is a part, 5 is the whole"



Children will need opportunities to look at and talk about different models and images as they move between representations.

Foundation Stage: F2
Subtraction

Key vocabulary:

Take (away), leave

How many are left/ left over?

How many have gone?

One less, two less, ten less

How many fewer is… than…?

Difference between

Is the same as

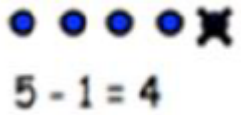
Strategies:

- Taking away ones
- Counting back
- Find the difference
- Part whole model
- Make 10

Guidance/ Models and Images

Children being with mostly pictorial representations
XXX take away 2 X [XX] = 1

Concrete apparatus us used to relate subtraction to taking away and counting how many objects are left
Children are encouraged to place objects in a row as a link to the bar model



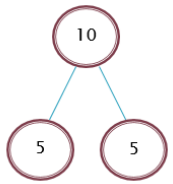
Construct number sentences verbally or using cards to match practical activities

Children are encouraged to read number sentences aloud in different ways:
“Five subtract one leaves four” “Four is equal to five subtract one”

Children make a record in pictures, words or symbols

Solve simple problems using fingers

Use bead strings to support the introduction of number bonds –
“how many beads can you see?”



Show subtraction on the part, part, whole model – “2 is a part, 3 is a part, 5 is the whole”
Children will need opportunities to look at and talk about different models and images as they move between representations.

Development Matters in the Early Years Foundation Stage (EYFS)

This non-statutory guidance material supports practitioners in implementing the statutory requirements of the EYFS

22 – 36 months	30 – 50 months	40 – 60 months	Early Learning Goal for Numbers
Creates and experiments with symbols and marks representing ideas of number. Begins to make comparisons between quantities. Uses some language of quantities, such as *'more'* and *'a lot'.* Knows that a group of things changes in quantity when something is added or taken away.	Beginning to represent numbers using fingers, marks on paper or pictures. Compares two groups of objects, saying when they have the same number. Separates a group of three or four objects in different ways, beginning to recognise that the total is still the same.	Says the number that is one more than a given number. Finds one more or one less from a group of up to five objects, then ten objects. In practical activities and discussion, beginning to use the vocabulary involved in adding and subtracting. Records, using marks that they can interpret and explain.	**Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. They solve problems, including doubling, halving and sharing.**

The importance of language development in FS mathematics **The sequential development of a childs language and vocabulary has a direct effect on their ability to explain their understanding to others. In terms of mathematical calculations a child also has to develop subject specific vocabulary alongside the development of their understanding of calculation concepts. These include:**
Recite number names in sequence (22-36 months) Uses number names and number language spontaneously (30-50months) Uses some number names accurately in play (30-50 months) Recites numbers in order to 10 (30-50 months)

Year 1
Addition

Pupils should be taught to:

- Read, write and interpret mathematical statements involving addition (+) and equals signs (=)
- Represent and use number bonds within 20
- Add one-digit and two-digit numbers to 20, including zero
- Solve one step problems that involve addition, using concrete objects and pictorial representations, and missing number problems

Key vocabulary:

Add, more, make, +, plus, sum, total, altogether, double, near double, one more, two more, ten more

How many more to make …?

How many more is… than…?

How much more is…?

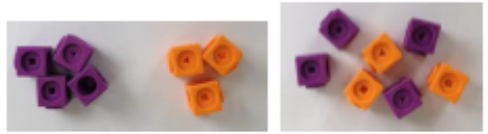
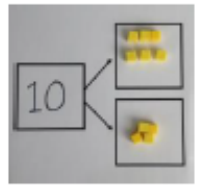
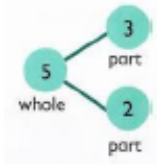
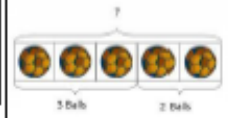
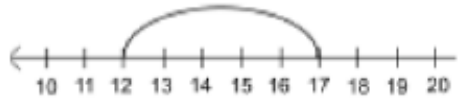
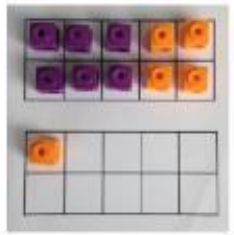
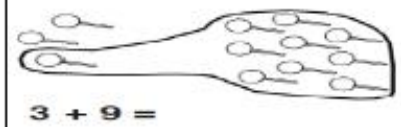
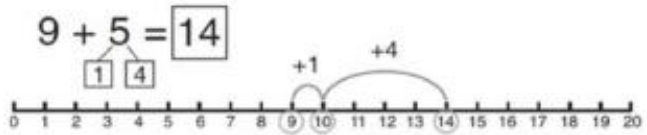
=, Equals sign, is the same as

Strategies:

- Combining two parts to make a whole: part whole model
- Starting and the bigger number and counting on
- Regrouping to make 10

Mental Strategies:

- Children should experience regular counting on and back from different numbers in 1s and in multiples of 2, 5 and 10
- Children should memorise and reason with number bonds for numbers to 20, experiencing the = sign in different positions
- They should see addition and subtraction as related operations
- Children should begin to understand addition as combining groups and counting on

Objective and Strategies	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part-whole model	  Use cubes to add two numbers together as a group or in a bar. 	   Use pictures to add two numbers together as a group or in a bar. 	4 + 3 = 7 10= 6 + 4  Use the part-part whole diagram as shown above to move into the abstract.
Starting at the bigger number and counting on	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	12 + 5 = 17  Start at the larger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.
Regrouping to make 10.	 6 + 5 = 11  Start with the bigger number and use the smaller number to make 10.	 Use pictures or a number line. Regroup or partition the smaller number to make 10. 	7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I add on now?

Year 2
Addition

Pupils should be taught to:

- Solve problems with addition: using concrete objects and pictorial representations, including those involving numbers, quantities and measures and apply their increasing knowledge of mental and written methods
- Recall and use addition facts to 20 fluently, and derive and use related facts up to 100
- Add numbers using concrete objects, pictorial representations and mentally including a two-digit number and ones, a two-digit number and tens, two two-digit numbers and three one-digit numbers
- Show that addition of two numbers can be done in any order (commutative)
- Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems

Key vocabulary:

Add, more, make, +, plus, addition, sum, total, altogether, double, near double, one more, two more, ten more, one hundred more

How many more to make …?

How many more is… than…?

How much more is…?

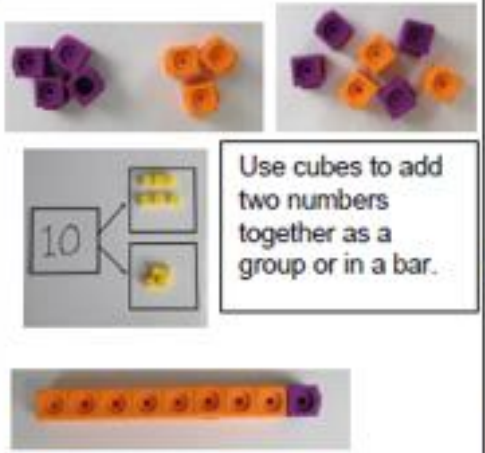
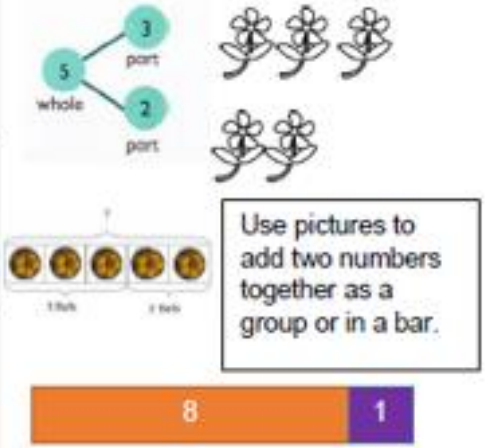
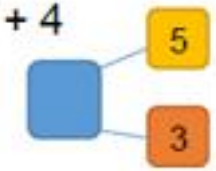
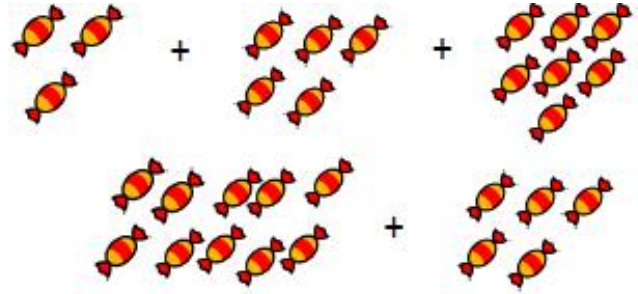
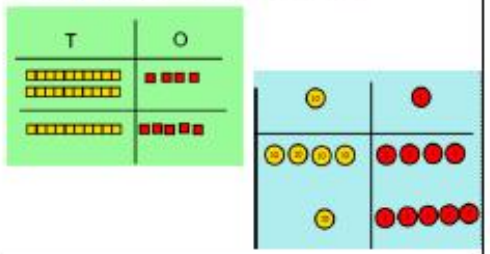
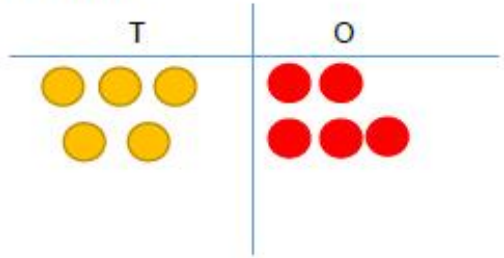
=, Equals sign, is the same as, tens boundary

Strategies:

- Combining two parts to make a whole: part whole model
- Starting and the bigger number and counting on
- Regrouping to make 10
- Adding three single digits
- Column method – no regrouping

Mental Strategies:

- Children should count regularly, on and back, in steps of 2, 3, 5 and 10. Counting forwards in tens from any number should lead to adding multiples of 10
- Number lines should be used to support mathematical thinking
- Children should practise addition to 20 to become increasingly fluent
- Children should learn to check their calculations by using the inverse
- They should continue to see addition as both combining groups and counting on

Objective and Strategies	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part-whole model	 Use cubes to add two numbers together as a group or in a bar.	 Use pictures to add two numbers together as a group or in a bar.	4 + 3 = 7 10= 6 + 4  Use the part-part whole diagram as shown above to move into the abstract.
Adding three single digits	4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7.  Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	 Add together three groups of objects. Draw a picture to recombine the groups to make 10.	4 + 7 + 6 = 10 + 7 10 = 17 Combine the two numbers that make 10 and then add on the remainder.
Column method- no regrouping	24 + 15= Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters. 	After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions. 	Calculations 21 + 42 = 21 + 42

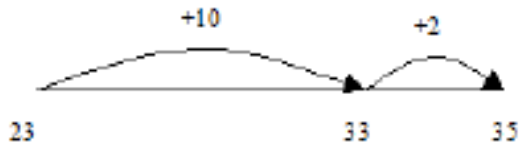
Written Methods Continued – Year 2

Missing number problems e.g 14 + 5 = 10 + □ 32 + □ + □ = 100 35 = 1 + □ + 5

It is valuable to use a range of representations (also see Y1). Continue to use numberlines to develop understanding of:

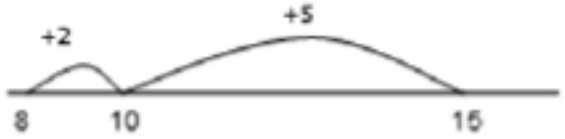
Counting on in tens and ones

23 + 12 = 23 + 10 + 2
= 33 + 2
= 35



Partitioning and bridging through 10.

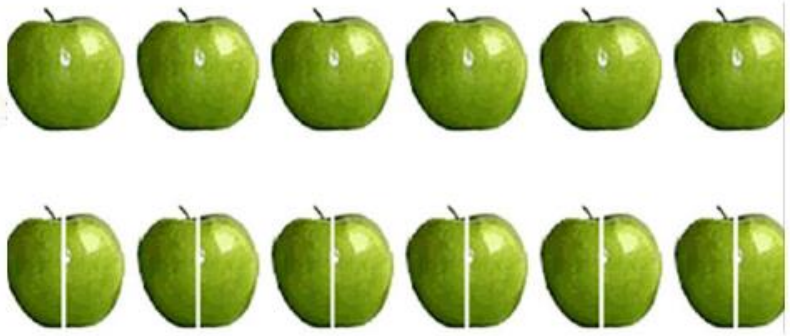
The steps in addition often bridge through a multiple of 10
e.g. Children should be able to partition the 7 to relate adding the 2 and then the 5.



8 + 7 = 15

Fractions

Counting in fractions up to 10, starting from any numbers using halves visually and on a number line



Year 3
Addition

Pupils should be taught to:

- Add numbers mentally, including: a three-digit number and ones, a three-digit number and tens, a three-digit number and hundreds
- Add numbers with up to three digits, using formal written methods of columnar addition
- Estimate the answer to a calculation and use inverse operations to check answers
- Solve problems, including missing number problems, using number facts, place value, and more complex addition

Key vocabulary:

Add, more, make, +, plus, addition, sum, total, altogether, double, near double, one more, two more, ten more, one hundred more

How many more to make …?

How many more is… than…?

How much more is…?

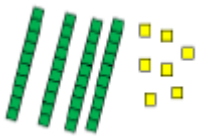
=, Equals sign, is the same as, tens boundary, hundreds boundary,

Strategies:

- Column method – regrouping (up to 3 digits)

Mental Strategies:

- Children should continue to count regularly, on and back, now including multiples of 4, 8, 50 and 100, and steps of 1/10
- The number line should continue to be used as an important image to support thinking, and the use of informed jottings should be encouraged
- Children should continue to partition numbers in different ways
- They should be encouraged to choose the mental strategies which are most efficient for the numbers involved
- Manipulatives can be used to support mental imagery and conceptual understanding. Children need to be shown how these images are related – what's the same? What's different?



Written Methods

Missing number problems using a range of equations as in Year 1 and 2 but with appropriate, larger numbers.

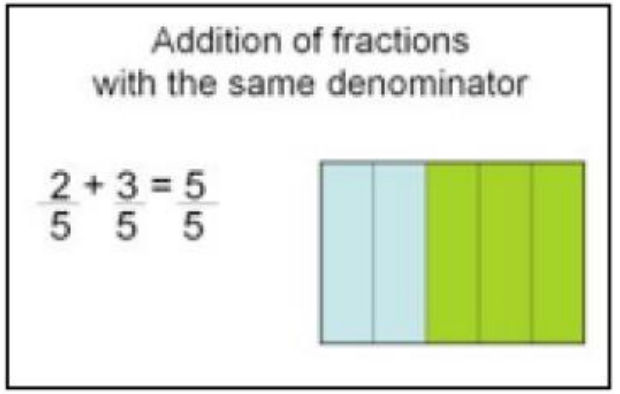
Partition in to tens and ones:
Partition both numbers and recombine.
Count on by partitioning the second number only e.g.

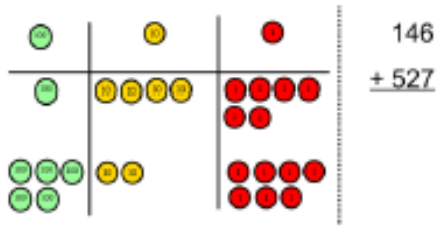
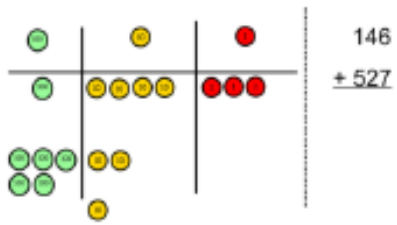
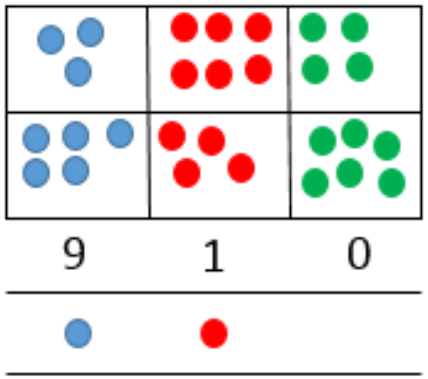
247 + 125 = 247 + 100 + 20 + 5
= 347 + 20 + 5
= 367 + 5
= 372

The bar model should continue to be used to support problem solving.

Fractions

Addition of fractions with the same denominator within one whole.



Objective and Strategies	Concrete	Pictorial	Abstract
Column method- regrouping	Make both numbers on a place value grid.  Add up the units and exchange 10 ones for one 10.  Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added. This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.	Children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding.  	Start by partitioning the numbers before moving on to clearly show the exchange below the addition. 20 + 5 40 + 8 60 + 13 = 73 536 + 85 621 11

Year 4
Addition

Pupils should be taught to:

- Add numbers with up to 4 digits using the formal written methods of columnar addition where appropriate
- Estimate and use inverse operations to check answers to a calculation
- Solve addition two-step problems in contexts, deciding which operations and methods to use and why

Key vocabulary:

Add, more, make, +, plus, increase, addition, sum, total, altogether, double, near double, one more, two more, ten more, one hundred more

How many more to make …?

How many more is… than…?

How much more is…?

=, Equals sign, is the same as, tens boundary, hundreds boundary,

Inverse

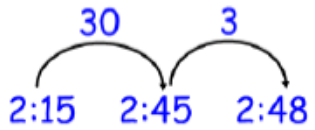
Strategies:

- Column method – regrouping (up to 4 digits)

Mental Strategies:

- Children should continue to count regularly, on and back, now including multiples of 6, 7, 9, 25 and 1000, and steps of 1/100
- The number line should continue to be used as an important image to support thinking, and the use of informed jottings should be encouraged where appropriate
- Children should continue to partition numbers in different ways
- They should be encouraged to choose from a range of mental strategies:

1. Counting forwards: 77 + 47, count on 40 from 77, then add 7
2. Reordering: 28 + 75 = 75 + 28
3. Partitioning: counting on or back: 5.6 + 3.7 -> 5.6 + 3 + 0.7 -> 8.6 + 0.7
4. Partitioning: compensating: 138 + 69 = 138 + 70 – 1
5. Partitioning: bridging through 60 to calculate a time interval – what was the time 33 minutes after 2.15pm



- Using known facts and place value to find related facts

Written Methods

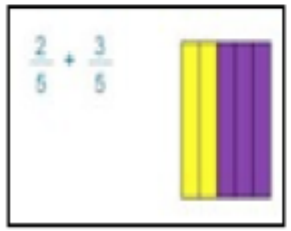
Missing number / digit problems:

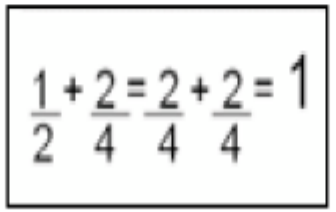
Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to support problem solving.

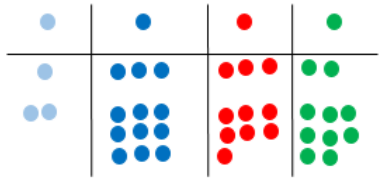
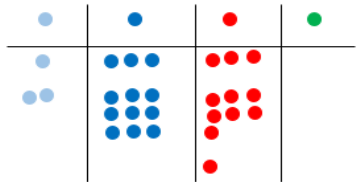
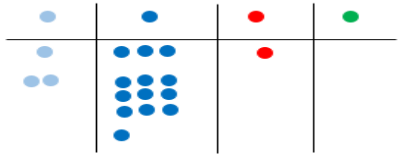
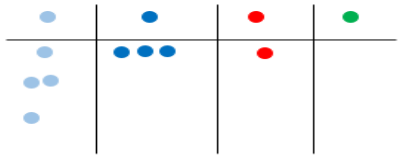
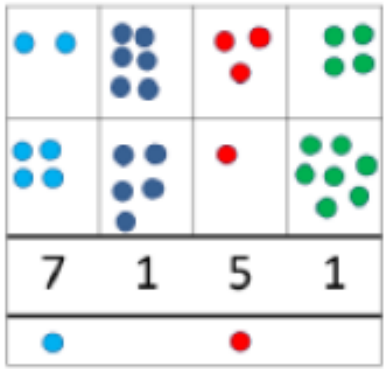
Fractions

Addition of fractions with the same denominator to become fluent through a variety of increasingly complex problems beyond one whole.

Counting using simple fractions and decimals, both forwards and backwards.





Objective and Strategies	Concrete	Pictorial	Abstract
Column method- regrouping	Make both numbers on a place value grid.  1372 +2938 Add up the units and exchange 10 ones for one 10.  Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.  	Children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding.  7 1 5 1 As children move on the decimals, money and decimal place value counters can be used to support learning.	Start by partitioning the numbers before moving on to clearly show the exchange below the addition. 20 + 5 40 + 8 60 + 13 = 73 As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here. 536 + 85 621 1 1 £23.59 £ 7.55 £31.14 1 1 1

Year 5
Addition

Pupils should be taught to:

- Add whole numbers with more than 4 digits, including using formal written methods (columnar addition)
- Add numbers mentally with increasingly large numbers
- Using rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- Solve addition multi-step problems in contexts, deciding which operations and methods to use and why

Key vocabulary:

Add, more, make, +, plus, increase, addition, sum, total, altogether, double, near double, one more, two more, ten more, one hundred more

How many more to make …?

How many more is… than…?

How much more is…?

=, Equals sign, is the same as, tens boundary, hundreds boundary, units boundary, tenths boundary

Inverse

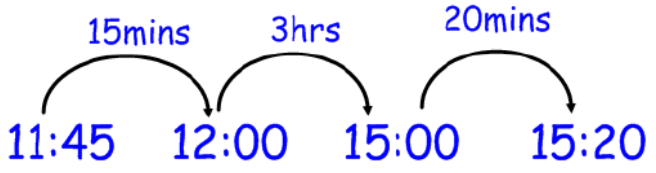
Strategies:

Column method – regrouping (with more than 4 digits) (Decimals – with the same amount of decimal places)

Mental Strategies:

- Children should continue to count regularly, on and back
- The number line should continue to be used as an important image to support thinking, and the use of informed jottings should be encouraged where appropriate
- Children should continue to partition numbers in different ways
- They should be encouraged to choose from a range of mental strategies:

1. Counting forwards and backwards in tenths and hundredths: 1.7 + 0.55
2. Reordering: 4.7 + 5.6 – 0.7 -> 4.7 – 0.7 + 5.6 -> 4 + 5.6
3. Partitioning: counting on or back: 540 + 280 = 540 + 200 + 80
4. Partitioning: bridging through multiples of 10
5. Partitioning: compensating: 5.7 + 3.9 = 5.7 + 4.0 – 0.1
6. Partitioning: bridging through 60 to calculate a time interval: It is 11.45, how many hours and minutes is it to 15.20?



- Using known facts and place value to find related facts

Written Methods

Missing number / digit problems:

Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to support problem solving. Children should practise with increasingly large numbers to aid fluency.

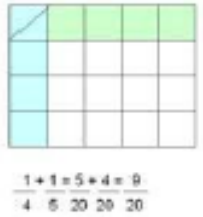
e.g. 12462 + 2300 = 14762

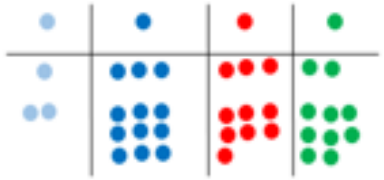
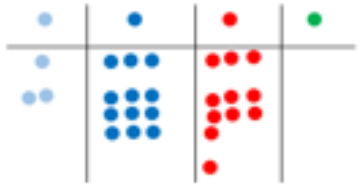
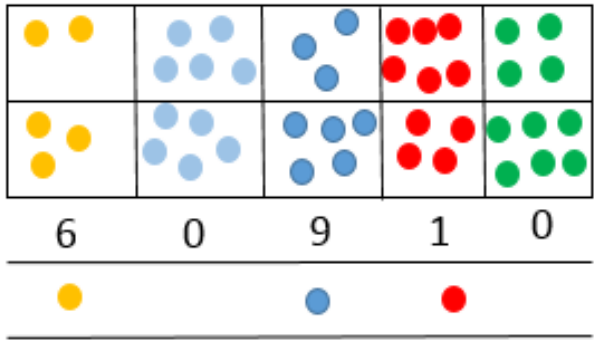
Focus on what they notice about the digits changing as they add different numbers.

Fractions

Add fractions with the same denominator and denominators that are multiples of the same number (to become fluent through a variety of increasingly complex problems and add fractions that exceed 1 as a mixed number)

$\frac{1}{2} + \frac{3}{4} = \frac{2}{4} + \frac{3}{4} = \frac{5}{4}$



Objective and Strategies	Concrete	Pictorial	Abstract
Column method- regrouping	Make both numbers on a place value grid.  1372 +2938 Add up the units and exchange 10 ones for one 10.  Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added. 	Children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding.  6 0 9 1 0	5362 +2859 8221 111 As the children move on, introduce decimals with the same number of decimal places and different. Money can be used. 23.59 47.55 71.14 11 1

Year 6
Addition

Pupils should be taught to:

- Perform mental calculations, including with mixed operations and large numbers
- Use their knowledge of the order of operations to carry out calculations involving the four operations
- Solve addition multi-step problems in contexts, deciding which operations and methods to use and why

Key vocabulary:

Add, more, make, +, plus, increase, addition, sum, total, altogether, double, near double, one more, two more, ten more, one hundred more

How many more to make …?

How many more is… than…?

How much more is…?

=, Equals sign, is the same as, tens boundary, hundreds boundary, units boundary, tenths boundary

Inverse

Strategies:

Column method – regrouping (Decimals – with the same and different amount of decimal places)

Mental Strategies:

- Consolidate previous years
- Perform mental calculations, including with mixed operations and larger numbers, using and practising a range of mental strategies
- Children should experiment with order of operations, investigating the effect of positioning brackets in different places: $20 – 5 \times 3 = 5$; $(20 – 5) \times 3 = 45$

Written Methods

Missing number/digit problems:

Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving.

Written methods

As year 5, progressing to larger numbers, aiming for both conceptual understanding and procedural fluency with columnar method to be secured. Continue calculating with decimals, including those with different numbers of decimal places

Problem Solving

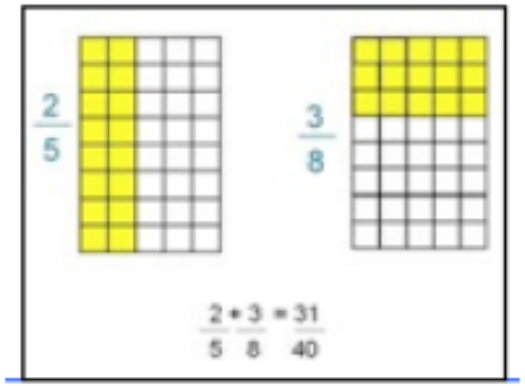
Teachers should ensure that pupils have the opportunity to apply their knowledge in a variety of contexts and problems (exploring cross curricular links) to deepen their understanding

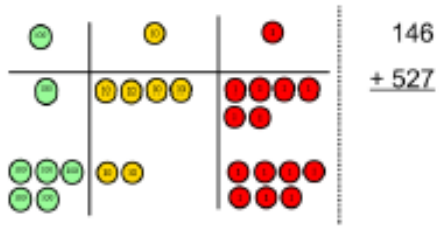
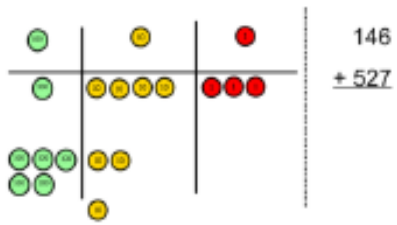
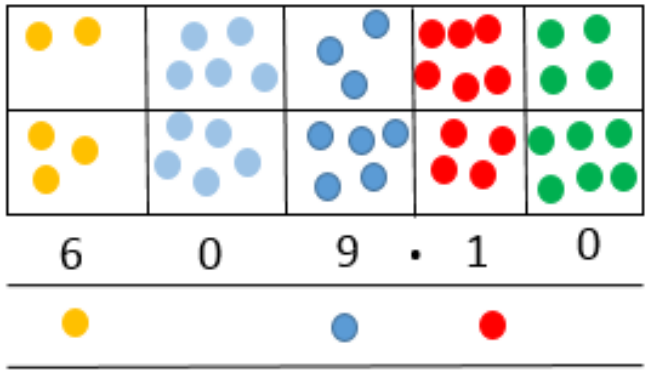
Fractions

Add fractions with different denominators and mixed numbers, using the concept of equivalent fractions.

Start with fractions where the denominator of one fraction is a multiple of the other (e.g. ½ + 1/8 = 5/8) and progress to varied and increasingly complex problems.

Practice calculations with simple fractions and decimal equivalents to aid fluency.



Objective and Strategies	Concrete	Pictorial	Abstract
Column method- regrouping	Make both numbers on a place value grid.  Add up the units and exchange 10 ones for one 10.  Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added. This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100. As children move on to decimals, money and decimal place value counters can be used to support learning.	Children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding.  	5362 + 5859 11221 1111 Introduce decimals with a different amount of decimal places. 34619.73 49156.50 83776.23 1 1 1

Year 1
Subtraction

Pupils should be taught to:

- Read, write and interpret mathematical statements involving subtraction (-) and equals (=) signs
- Represent and use related subtraction facts within 20
- Subtract one-digit and two-digit numbers to 20, including zero
- Solve one-step problems that involve subtraction, using concrete objects and pictorial representations, and missing number problems

Key vocabulary:

-, subtract, take (away), minus, leave

How many are left/ left over?

How many have gone?

One less, two less, ten less

How many fewer is… than…?

How much less is…?

Difference between

Half, halve

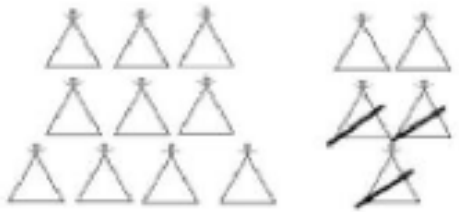
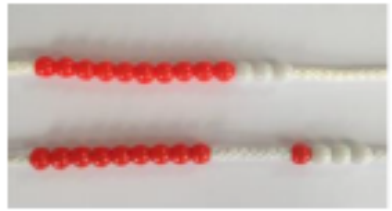
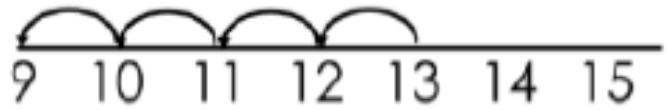
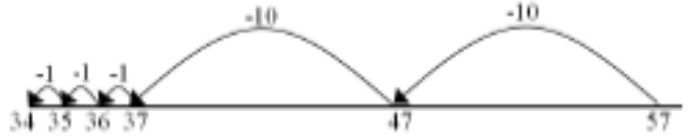
=, equals sign, Is the same as

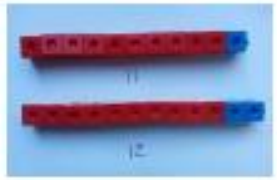
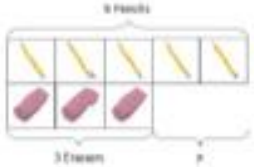
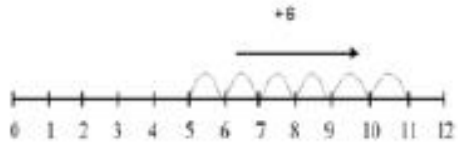
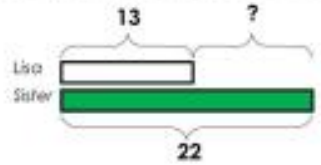
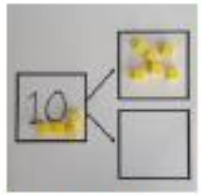
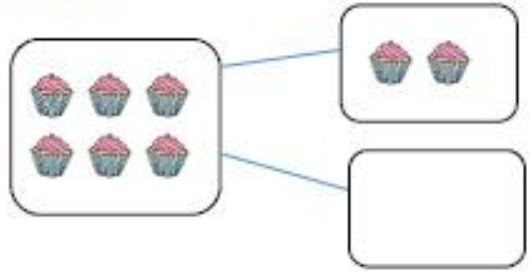
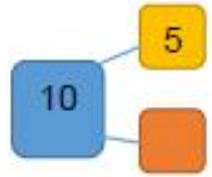
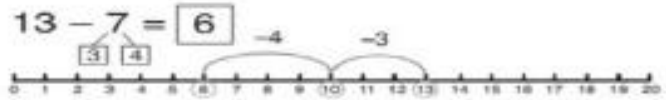
Strategies:

- Taking away ones
- Counting back
- Find the difference
- Part whole model
- Make 10

Mental Strategies:

- Memorise and reason with number bonds

Objective and Strategies	Concrete	Pictorial	Abstract
Taking away ones	Use physical objects, counters, cubes etc to show how objects can be taken away.  6 – 2 = 4  	Cross out drawn objects to show what has been taken away.  15 – 3 = 12	18 -3= 15 8 – 2 = 6
Counting back	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.  13 – 4 Use counters and move them away from the group as you take them away counting backwards as you go. 	Count back on a number line or number track  9 10 11 12 13 14 15 Start at the bigger number and count back the smaller number showing the jumps on the number line.  -10 -10 -1 -1 -1 34 35 36 37 47 57 This can progress all the way to counting back using two 2 digit numbers.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help.

Objective and Strategies	Concrete	Pictorial	Abstract
Find the difference	Compare amounts and objects to find the difference.  Use cubes to build towers or make bars to find the difference  Use basic bar models with items to find the difference	 Count on to find the difference. Draw bars to find the difference between 2 numbers. **Comparison Bar Models** *Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.* 	Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.
Part Part Whole Model	 Link to addition- use the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 =	Use a pictorial representation of objects to show the part part whole model. 	 Move to using numbers within the part whole model.
Make 10	14 – 9 =  Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.	 Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.	16 – 8= How many do we take off to reach the next 10? How many do we have left to take off?

Year 2
Subtraction

Pupils should be taught to:

- Solve problems with subtraction using concrete objects and pictorial representations, including those involving numbers, quantities and measures
- Applying their increasing knowledge of mental and written methods
- Recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100
- Subtract numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones, a two-digit number and tens, two two-digit numbers and adding three one-digit numbers
- Show that subtraction of one number from another cannot be done in any order
- Use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems

Key vocabulary:

-, subtract, subtraction, take (away), minus, leave

How many are left/ left over?

How many have gone?

One less, two less, ten less, one hundred less

How many fewer is… than…?

How much less is…?

Difference between

Half, halve

=, equals sign, Is the same as

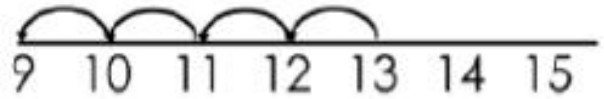
Tens boundary

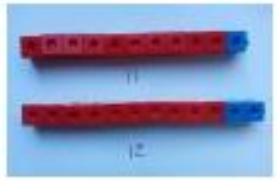
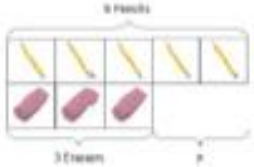
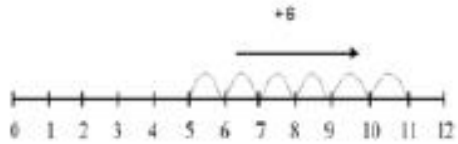
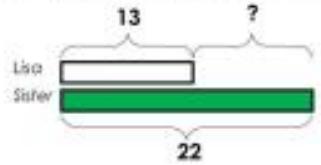
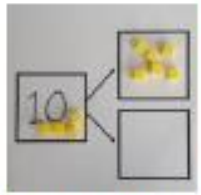
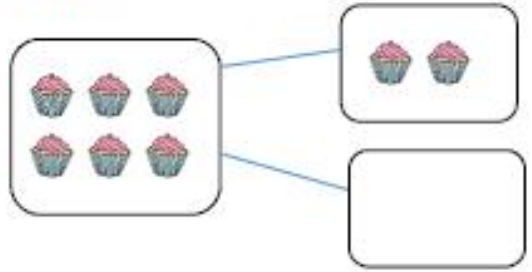
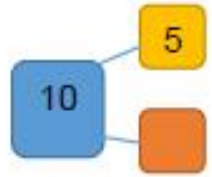
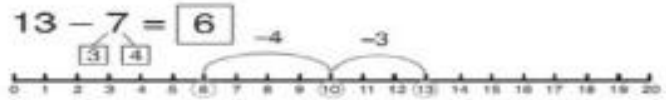
Strategies:

- Counting back
- Find the difference
- Part whole model
- Make 10
- Column method no regrouping

Fractions:

- Children could count in fractions up to 10, in halves

Objective and Strategies	Concrete	Pictorial	Abstract
Counting back	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.  13 – 4 Use counters and move them away from the group as you take them away counting backwards as you go. 	Count back on a number line or number track  Start at the bigger number and count back the smaller number showing the jumps on the number line.  This can progress all the way to counting back using two 2 digit numbers.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help.

Objective and Strategies	Concrete	Pictorial	Abstract
Find the difference	Compare amounts and objects to find the difference.  Use cubes to build towers or make bars to find the difference  Use basic bar models with items to find the difference	 Count on to find the difference. Draw bars to find the difference between 2 numbers. **Comparison Bar Models** *Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.* 	Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.
Part Part Whole Model	 Link to addition- use the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 =	Use a pictorial representation of objects to show the part part whole model. 	 Move to using numbers within the part whole model.
Make 10	14 – 9 =  Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.	 Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.	16 – 8= How many do we take off to reach the next 10? How many do we have left to take off?

Year 3
Subtraction

Pupils should be taught to:

- Subtract numbers mentally, including: a three-digit number and ones, a three-digit number and tens and a three digit number and hundreds
- Subtract numbers with up to three digits, using formal written methods of columnar subtraction
- Estimate the answer to a calculation and use inverse operations to check answers
- Solve problems, including missing number problems, using number facts, place value and more complex subtraction

Key vocabulary:

-, subtract, subtraction, take (away), minus, leave

How many are left/ left over?

How many have gone?

One less, two less, ten less, one hundred less

How many fewer is… than…?

How much less is…?

Difference between

Half, halve

=, equals sign, Is the same as

Tens boundary, hundreds boundary

Strategies:

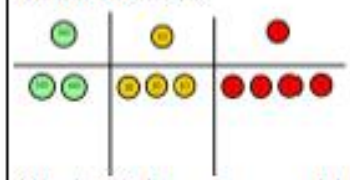
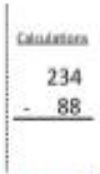
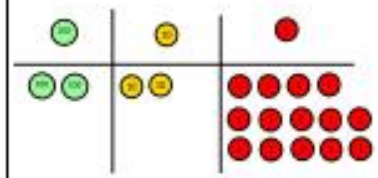
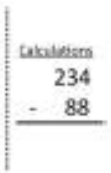
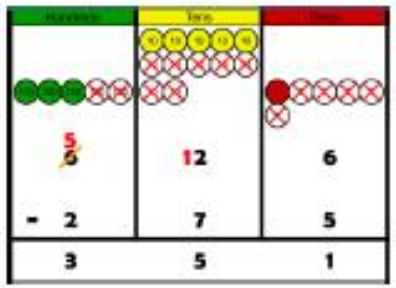
- Column method with regrouping (up to 3 digits)

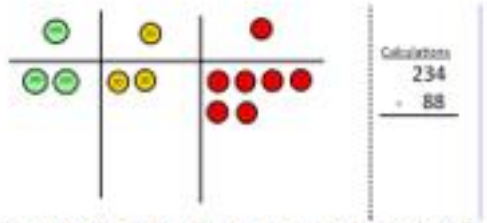
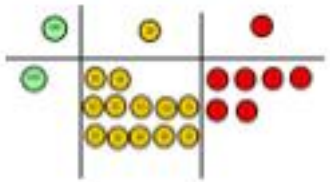
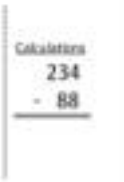
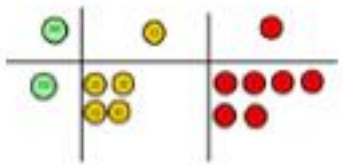
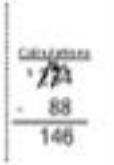
Mental Strategies:

- Children should continue to count regularly, on and back, now including multiples of 4, 8, 50 and 100, and steps of 1/10
- The number line should continue to be used as an important image to support thinking, and the use of informed jottings should be encouraged
- Children should continue to partition numbers in different ways
- They should be encouraged to choose the mental strategies which are most efficient for the numbers involved: counting up/ counting back

Fractions:

- Count up and down in tenths from any given number, including mixed numbers
- Subtract fractions with the same denominator within one whole

Objective and Strategies	Concrete	Pictorial	Abstract
Column method with regrouping	Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges. Make the larger number with the place value counters   Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.   	 Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.  When confident, children can find their own way to record the exchange/regrouping. Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.	 Children can start their formal written method by partitioning the number into clear place value columns.  Moving forward the children use a more compact method.

Objective and Strategies	Concrete
	Now I can subtract my ones.  Calculations 234 - 88 Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.   Calculations 234 - 88 Now I can take away eight tens and complete my subtraction   Calculations - 88 146 Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.

Year 4
Subtraction

Pupils should be taught to:

- Subtract numbers with up to four digits using the formal written methods of columnar subtraction where appropriate
- Estimate and use inverse operations to check answers to a calculation
- Solve subtraction two-step problems in contexts, deciding which operations and methods to use and why

Key vocabulary:

-, subtract, subtraction, take (away), minus, decrease, leave

How many are left/ left over?

How many have gone?

One less, two less, ten less, one hundred less

How many fewer is… than…?

How much less is…?

Difference between

Half, halve

=, equals sign, Is the same as

Tens boundary, hundreds boundary

Inverse

Strategies:

- Column method with regrouping (up to 4 digits)

Mental Strategies:

- Children should continue to count regularly, on and back, now including multiples of 6, 7, 9, 25 and 1000, and steps of 1/100
- The number line should continue to be used as an important image to support thinking, and the use of informed jottings should be encouraged where appropriate
- Children should continue to estimate answers and partition numbers in different ways
- They should be encouraged to choose from a range of mental strategies:

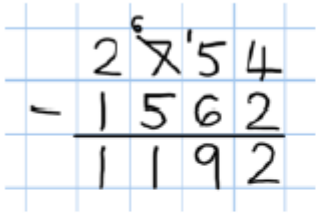
1. Counting forwards and backwards: 124 – 47, count back 40 from 124, then 4 to 80, then 3 to 77
2. Partitioning: compensating: 138 – 69 = 138 – 70 + 1
3. Partitioning: bridging through 60 to calculate a time interval – what was the time 33 minutes before 2.15pm?

- Using known facts and place value to find related facts

Written Methods

- Subtract numbers with up to 4 digits using the formal written methods of columnar subtraction where appropriate
- Use inverse operations to check answers to calculations
- Solve addition and subtraction problems in contexts deciding which methods to use (bar model, number line, columnar subtraction)

Compact column subtraction



Links to other strands:

- Identify, represent and estimate numbers using different representations. (*Place value*)
- Recognise the place value of each digit in a four digit number.
- Solve addition and subtraction two
- Estimate, compare and calculate different measures, including money in pounds and pence.

Fractions:

- Count up and down in hundredths from any given number, including mixed numbers
- Subtract fractions with the same denominator
- Solve simple measure and money problems involving fractions and decimals to two decimal places

Year 5
Subtraction

Pupils should be taught to:

- Subtract whole numbers with more than 4 digits, including using formal written methods (columnar subtraction)
- Subtract numbers mentally with increasingly large numbers
- Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- Solve subtraction multi-step problems in contexts, deciding which operations and methods to use and why

Key vocabulary:

-, subtract, subtraction, take (away), minus, decrease, leave

How many are left/ left over?

How many have gone?

One less, two less, ten less, one hundred less

How many fewer is… than…?

How much less is…?

Difference between

Half, halve

=, equals sign, Is the same as

Tens boundary, hundreds boundary, units boundary, tenths boundary

Inverse

Strategies:

Column method with regrouping (with more than 4 digits)

(Decimals with the same amount of decimal places)

Mental Strategies:

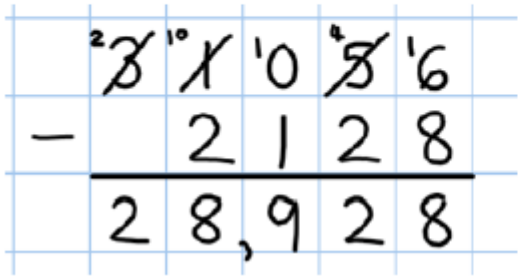
- Consolidate and decide upon appropriate mental strategies: find differences by counting up, partitioning, applying known facts, bridging through 10 and multiples of 10, subtracting 9/11 by compensating, counting on to or back from the largest number
- Add and subtract decimals, including a mix of whole numbers and decimals, decimals with different numbers of decimal places and complements of 1
- Add and subtracts tenths, and one-digit numbers and tenths
- Use appropriate mental strategies to solve problems involving time, money and measure

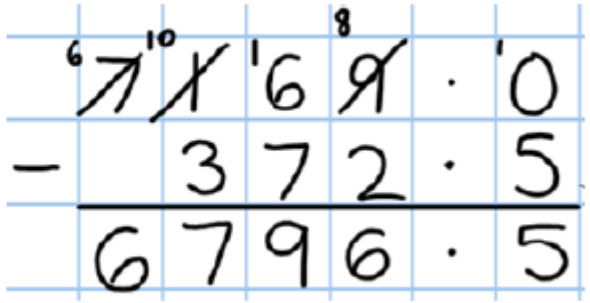
Written Methods:

- Subtract whole numbers with more than 4 digits, including using formal methods (columnar subtraction)
- Practise adding and subtracting decimals
- Use subtraction to solve problems involving time, money and measure using decimal notation (up to 3dp)

Compact column subtraction

(with 'exchanging').





Fractions:

- Subtract fractions with the same denominator and denominators that are multiples of the same number
- Solve problems involving numbers up to three decimal places
- Mentally add and subtract tenths, one-digit whole numbers and tenths

Year 6
Subtraction

Pupils should be taught to:

- Perform mental calculations, including with mixed operations and large numbers
- Use their knowledge of the order of operations to carry out calculations involving the four operations
- Solve subtraction multi-step problems in contexts deciding which operations and methods to use and why
- **Key vocabulary:**

-, subtract, subtraction, take (away), minus, decrease, leave

How many are left/ left over?

How many have gone?

One less, two less, ten less, one hundred less

How many fewer is… than…?

How much less is…?

Difference between

Half, halve

=, equals sign, Is the same as

Tens boundary, hundreds boundary

Inverse

Strategies:

Column method with regrouping

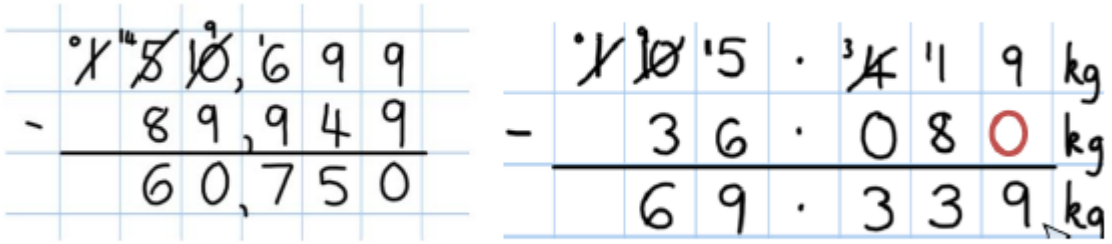
(Decimals with different amounts of decimal places)

Mental Strategies:

- Use estimation and inverse operations to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy
- Use inverse knowledge to solve calculations
- Undertake mental calculations with increasingly large numbers and more complex calculations
- Use appropriate mental strategies to solve problems involving time, money and measure of up to three decimal places where appropriate

Written Methods:

- Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)
- Solve problems involving the calculation and conversions of units of measure, time and money using decimal notation up to three decimal places where appropriate



Fractions:

- Subtract fractions with different denominators and mixed numbers
- Practise calculations with simple fractions and decimal fraction equivalents to aid fluency

Year 1
Multiplication

Pupils should be taught to:

- Solve one-step problems involving multiplication, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher

Key vocabulary:

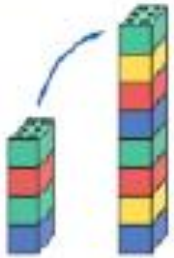
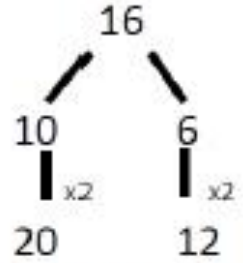
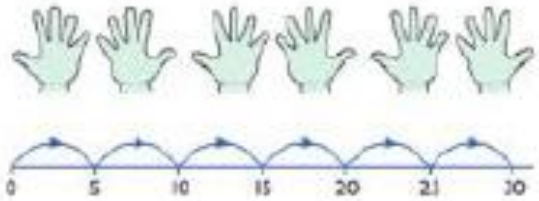
Double

Strategies:

- Double numbers to 10
- Counting in multiples of twos, fives and tens
- Arrays (with support)

Mental Strategies:

- Develop the language of multiplication by encouraging the children to explain what they are doing
- Children do not need to record number sentences using the symbols

Objective and Strategies	Concrete	Pictorial	Abstract
Doubling	Use practical activities to show how to double a number.  double 4 is 8 4 × 2 = 8	Draw pictures to show how to double a number. Double 4 is 8 	 Partition a number and then double each part before recombining it back together.
Counting in multiples	  Count in multiples supported by concrete objects in equal groups.	 Use a number line or pictures to continue support in counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30

Fractions

Start using vocabulary related to fractions e.g. half, quarter, whole. Relate fractions to sharing out and measures.

Respond to fractions in real life contexts, for example;

Half fill this jug.

Is this pot/ cylinder/ container / jug less/ more than / about half filled?

Shade $\frac{1}{2}$ of this rectangle, circle…

Fold this picture in half.

Cut half way along the line.

Can you split these sweets into two equal groups? You now have half each.

Look at the clock, the big hand is half way around the clock, it is at 6 and the little hand is half way between the 2 and 3 so the time is half past 3.

Year 2
Multiplication

Pupils should be taught to:

- Recall and use multiplication facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- Calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication (x) and equals (=) signs
- Show that multiplication of two numbers can be done in any order (commutative)
- Solve problems involving multiplication, using materials, arrays, repeated addition, mental methods, and multiplication, including problems in contexts

Key vocabulary:

Lots of, groups of, x, times, multiply, multiplied by, multiple of

Once, twice, three times, ten times

Times as (big, long, wide)

Repeated addition

Array

Row, column

Double

Strategies:

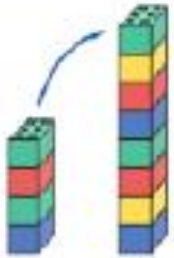
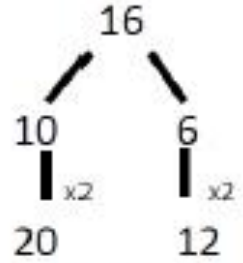
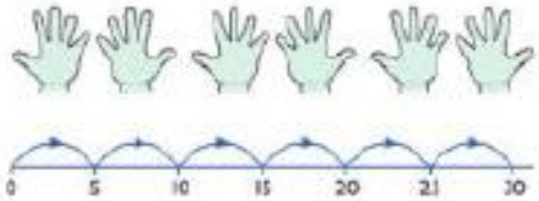
- Double numbers to 10 for rapid recall and use this knowledge to double larger numbers
- Counting in multiples
- Repeated addition
- Arrays – showing commutative multiplication

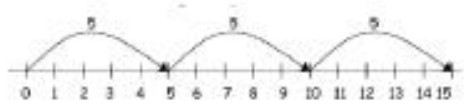
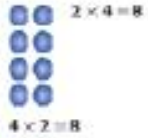
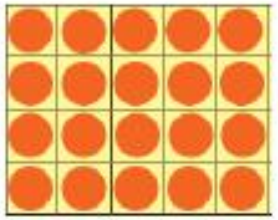
Mental Strategies:

- Rapid recall of multiplication facts – 2, 5, and 10 times tables
- Recognise odd and even numbers
- Show that multiplication of two numbers can be done in any order (commutativity)
- Use a variety of language to describe multiplication

Fractions

I have half, I have 10, how many were there?

Objective and Strategies	Concrete	Pictorial	Abstract
Doubling	Use practical activities to show how to double a number.  double 4 is 8 4 × 2 = 8	Draw pictures to show how to double a number. Double 4 is 8 	 Partition a number and then double each part before recombining it back together.
Counting in multiples	  Count in multiples supported by concrete objects in equal groups.	 Use a number line or pictures to continue support in counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30

Objective and Strategies	Concrete	Pictorial	Abstract
Repeated addition	   Use different objects to add equal groups.	There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?  2 add 2 add 2 equals 6  5 + 5 + 5 = 15	Write addition sentences to describe objects and pictures. 
Arrays- showing commutative multiplication	Create arrays using counters/ cubes to show multiplication sentences.  	Draw arrays in different rotations to find commutative multiplication sentences.    Link arrays to area of rectangles.	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15

Year 3
Multiplication

Pupils should be taught to:

- Recall and use multiplication facts for the 3, 4 and 8 multiplication tables
- Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods. Write them using the multiplication, division and equals signs
- Solve problems, including missing number problems, involving multiplication , positive integer scaling problems and correspondence problems in which n objects are connected to m objects

Key vocabulary:

Lots of, groups of, x, times, multiply, multiplication, multiplied by, multiple of, product

Once, twice, three times, ten times

Times as (big, long, wide)

Repeated addition

Array

Row, column

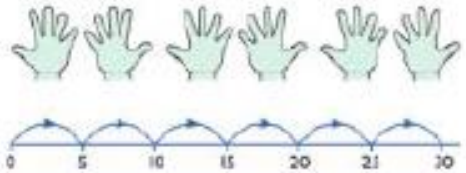
Double

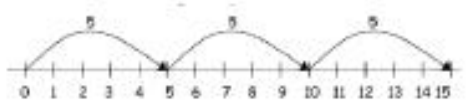
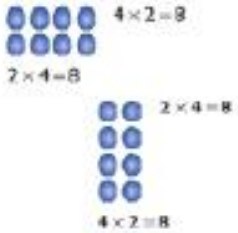
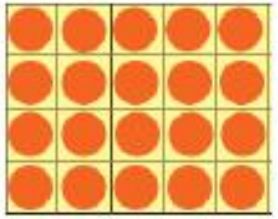
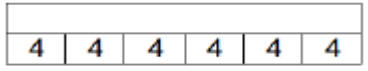
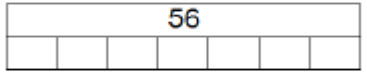
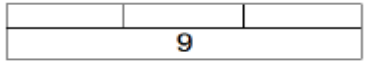
Strategies:

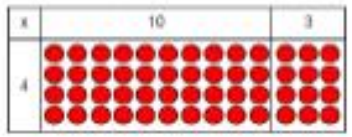
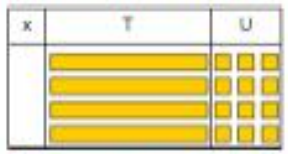
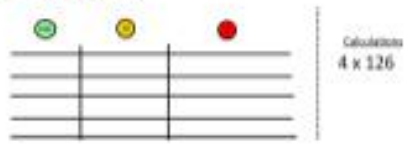
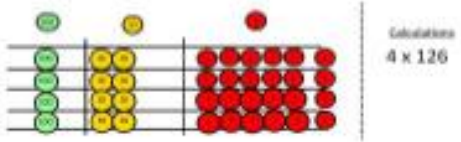
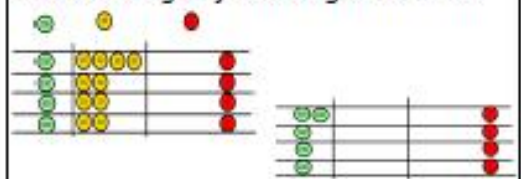
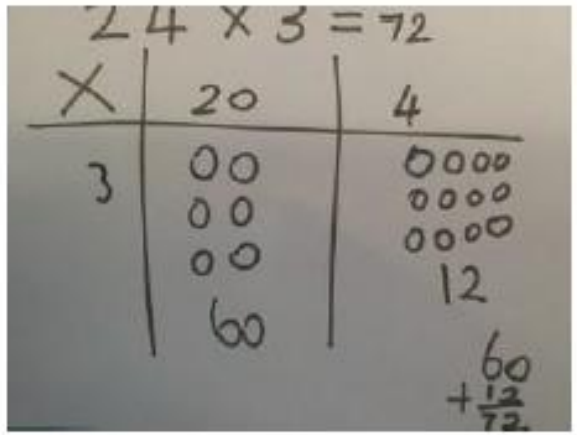
- Counting in multiples of 4, 8, 50 and 100
- Repeated addition
- Arrays showing commutative multiplication
- Grid method

Mental Strategies:

- Recall and use multiplication facts for the 3, 4 and 8 multiplication tables
- Use doubling to connect 2, 4, and 8 times tables
- Partitioning: multiply the tens first and then the ones
- Include missing number statements

Objective and Strategies	Concrete	Pictorial	Abstract
Counting in multiples	  Count in multiples supported by concrete objects in equal groups.	 Use a number line or pictures to continue support in counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30

Objective and Strategies	Concrete	Pictorial	Abstract
Repeated addition	   Use different objects to add equal groups.	There are 3 plates. Each plate has 2 star biscuits on them. How many biscuits are there?  2 add 2 add 2 equals 6  5 + 5 + 5 = 15	Write addition sentences to describe objects and pictures. 
Arrays- showing commutative multiplication	Create arrays using counters/ cubes to show multiplication sentences.  	Draw arrays in different rotations to find commutative multiplication sentences.   Link arrays to area of rectangles.	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15
		   Bar modelling can be used to support children with problem solving	

Objective and Strategies	Concrete	Pictorial	Abstract
Grid Method	Show the link with arrays to first introduce the grid method.  4 rows of 10 4 rows of 3 Move on to using Base 10 to move towards a more compact method.  4 rows of 13 Move on to place value counters to show how we are finding groups of a number.We are multiplying by 4 so we need 4 rows.  Fill each row with 126.  Add up each column, starting with the ones making any exchanges needed.  Then you have your answer.	Children can represent the work they have done with place value counters in a way that they understand. They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.  	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. | × | 30 | 5 | | 7 | 210 | 35 | 210 + 35 = 245 Linking 7 x 3 = 21 7 x 30 = 210 Ten times bigger

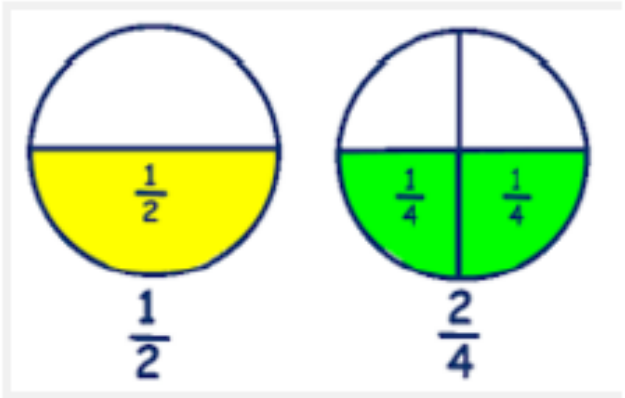
Fractions

recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators

⅓ =

⅚ =

recognise and show, using diagrams, equivalent fractions with small denominators



Year 4
Multiplication

Pupils should be taught to:

- Recall multiplication facts for multiplication tables up to 12 x 12
- Use place value, known and derived facts to multiply mentally, including: multiplying by 0 and 1 and multiplying together three numbers
- Recognise and use factor pairs and commutativity in mental calculations
- Multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- Multiply together 3 numbers
- Solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects

Key vocabulary:

Lots of, groups of, x, times, multiply, multiplication, multiplied by, multiple of, product

Once, twice, three times, ten times

Times as (big, long, wide)

Repeated addition

Array

Row, column

Double

Strategies:

- Column multiplication (2 and 3 digit multiplied by 1 digit)
- Count in multiples of 6, 7, 9, 25 and 1000

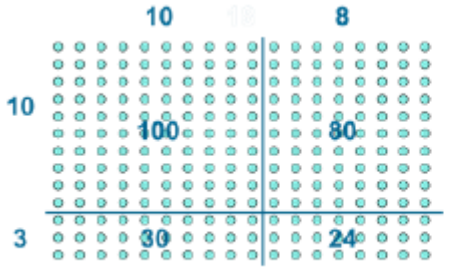
Mental Strategies:

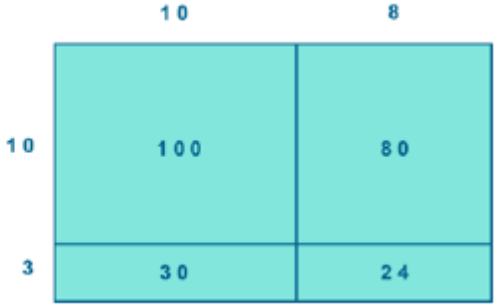
- Children count regularly no including multiples of 6, 7, 9, 25 and 1000, and steps of 1/100
- Become fluent and confident to recall all times tables to x12
- Multiply three numbers together
- Partitioning using x10 x20
- Doubling to solve x2 x4 x8
- Use of commutativity

Written Methods:

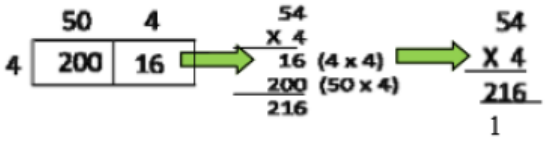
(Progressing to 3digit x 2digit)

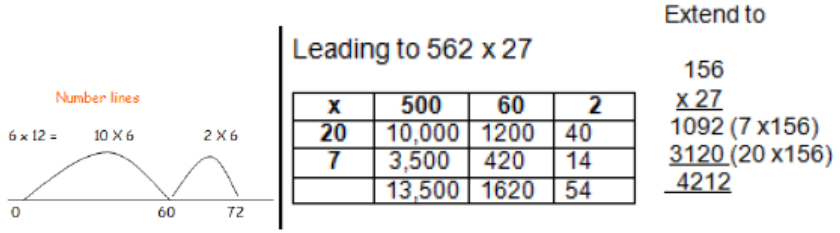
Children to embed and deepen their understanding of the grid method to multiply 2d x 2d. Ensure this is still linked back to their understanding of arrays and place value counters.





- Multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- Estimate before calculating
- Ensure written methods build on/ relate to mental methods
- Introduce alongside grid and expanded column methods



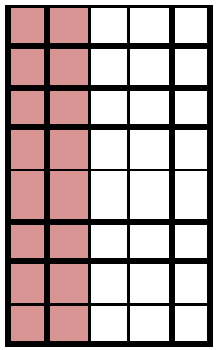


Fractions

- **recognise and show, using diagrams, families of common equivalent fractions**
- understand the relation between non-unit fractions and multiplication and division of quantities, with particular emphasis on tenths and hundredths.
- make connections between fractions of a length, of a shape and as a representation of one whole or set of quantities.
- use factors and multiples to recognise equivalent fractions and simplify where appropriate

$$\frac{4}{10}\ \frac{6}{15}\ \frac{8}{20}\ \frac{10}{25}\ \frac{12}{30}\ \frac{14}{35}\ \frac{16}{40}$$

$$\frac{2}{5} = \frac{16}{40}$$



Year 5
Multiplication

Pupils should be taught to:

- Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- Know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- Establish whether a number up to 100 is prime and recall prime numbers up to 19
- Multiply numbers up to 4 digits by a one or two-digit number using a formal written method, including long multiplication for two-digit numbers
- Multiply numbers mentally drawing upon known facts
- Multiply whole numbers and those involving decimals by 10, 100 and 1000

Key vocabulary:

Lots of, groups of, x, times, multiply, multiplication, multiplied by, multiple of, product

Once, twice, three times, ten times

Times as (big, long, wide)

Repeated addition

Array

Row, column

Double

Strategies:

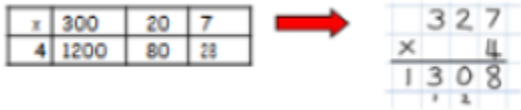
Column multiplication (up to 4 digit numbers multiplied by 1 or 2 digits)

Mental Strategies:

- Children continue to count regularly
- Multiply by 10, 100, 1000, including decimals
- Partitioning using x10 x20
- Doubling to solve x2 x4 x8
- Recall of times tables
- Use of commutativity
- If children know the times tables facts to 12 x 12, can they use this to recite other times tables? (13 / 24 times tables)

Written Methods:

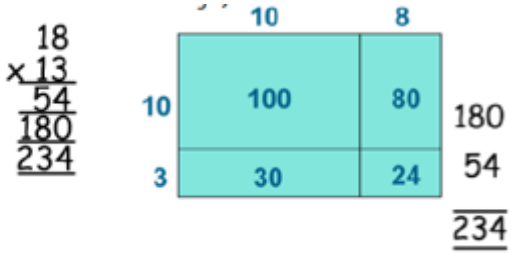
Short multiplication for multiplying by one digit



- Show children the link from the grid method

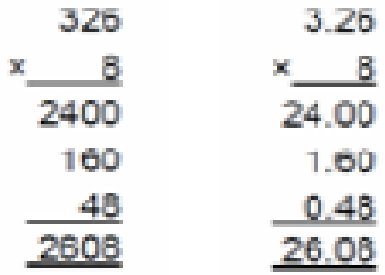
Long multiplication for multiplying by two digits

- Long multiplication using place value counters. Children explore how the grid method supports an understanding of long multiplication (2digit x 2 digit)



Multiply numbers up to 4 digits by a one or two digit number using a formal written method including long multiplication

- Compact methods for multiplication are efficient but often do not make the value of each digit explicit. When introducing multiplication of decimals, it is sensible to take children back to an expanded form such as the grid method where the value of each digit is clear, to ensure that children understand the process.
- Demonstrate multiplication of a decimal number alongside its whole number equivalent



Fractions

- Multiply proper fractions by mixed numbers and whole numbers supported my materials and diagrams
- Identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths.

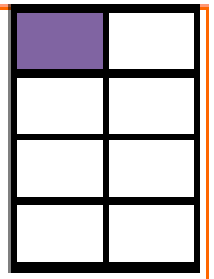
¼ x ½

Scaling by ½

"finding a half of a quarter"

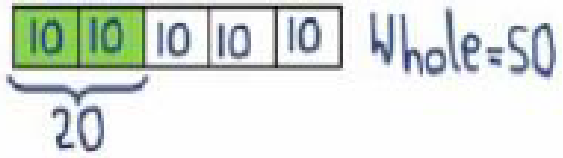
½ x ¼

"¼ of a ½": find a ½, then divide it by 4.



Encourage children to draw diagrams to represent situations or problems involving fractions
Model how to do this, for example:

$\frac{2}{5}$ of a number is 20. What is the number?



Year 6
Multiplication

Pupils should be taught to:

- Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- Perform mental calculations, including with mixed operations and larger numbers
- Identify common factors, common multiples and prime numbers
- Use their knowledge of the order of operations to carry out calculations involving the four operations

Key vocabulary:

Lots of, groups of, x, times, multiply, multiplication, multiplied by, multiple of, product

Once, twice, three times, ten times

Times as (big, long, wide)

Repeated addition

Array

Row, column

Double

Strategies:

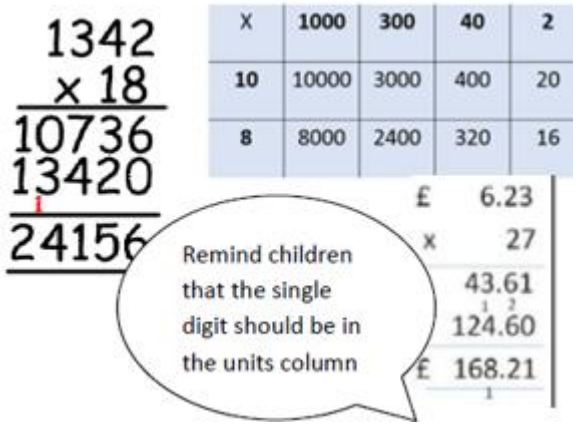
Column multiplication (multi digit up to 4 digits by a 2 digit number)

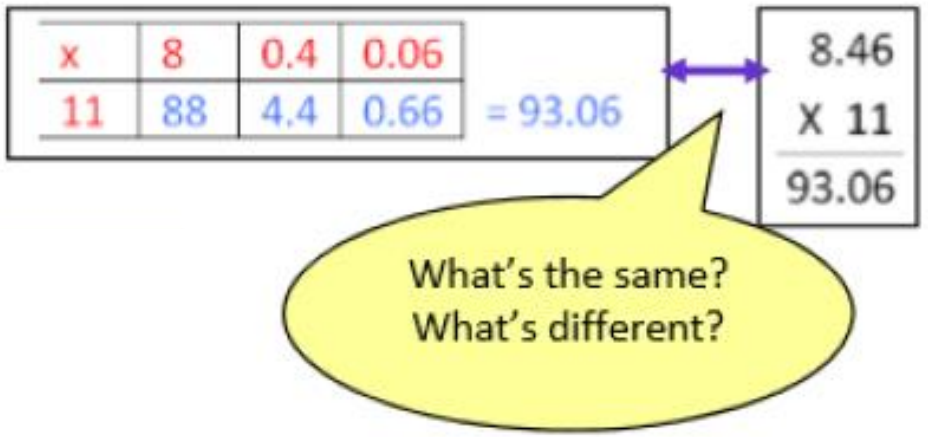
Mental Strategies

- Use all the multiplication tables to calculate mathematical statements in order to maintain fluency
- Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy
- Children should know the square numbers up to 12 x 12 and derive the corresponding squares of multiples of 10

Written Methods:

- Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication (short and long multiplication)
- Multiply one-digit numbers with up to two decimal places by whole numbers
- Look at long multiplication calculations containing errors, identify the errors and determine how they should be corrected

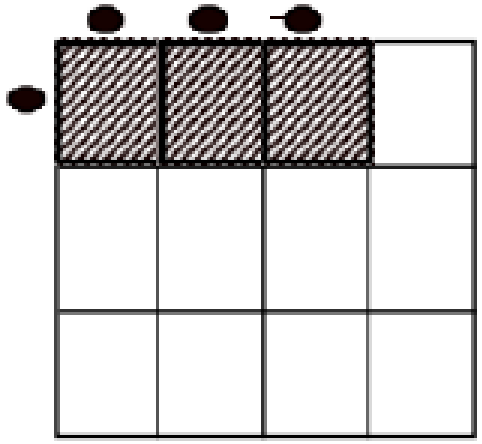




Fractions

Multiplying fractions

$\frac{1}{3} \times \frac{3}{4} = \frac{3}{12}$



$\frac{2}{3} \times \frac{3}{5} = \frac{6}{15}$

Year 1
Division

Pupils should be taught to:

- Solve one-step problems involving division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Key vocabulary:

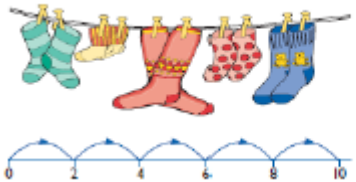
Halve, share, left over

Strategies:

- Sharing objects into groups
- Division as grouping

Mental Strategies:

- Children should experience regular counting on and back from different numbers in 1s and multiples of 2, 5 and 10
- They should begin to recognise the number of groups counted to support understanding of relationship between multiplication and division



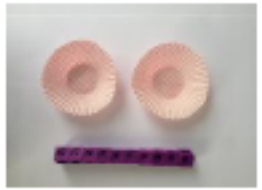
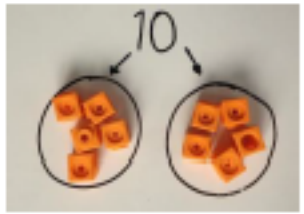
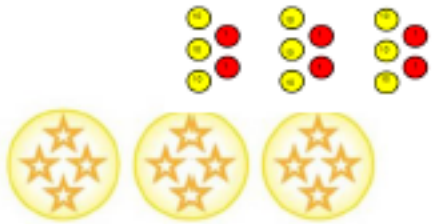
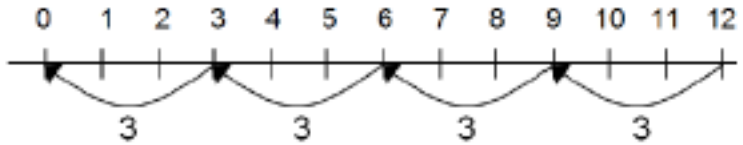
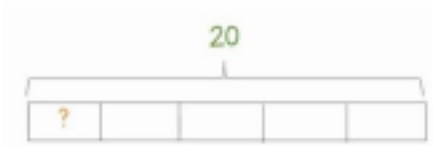
$2+2+2+2+2=10$
$2\times5=10$
2 multiplied by 5
5 pairs
5 hops of 2

- Children should begin to understand division as both sharing and grouping.

Sharing - 6 sweets shared between 2 people, how many do they have each?

Grouping – how many 2's are in 6?

- They should use objects to group and share amounts to develop understanding of division in a practical sense
- Children should begin to explore finding simple fractions of objects, numbers and quantities

Objective and Strategies	Concrete	Pictorial	Abstract
Sharing objects into groups	   I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  8 ÷ 2 = 4	Share 9 buns between three people. 9 ÷ 3 = 3
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.   96 ÷ 3 = 32 	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.  20 ÷ 5 = ? 5 x ? = 20	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?

Year 2
Division

Pupils should be taught to:

- Recall and use division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- Calculate mathematical statements for division within the multiplication tables and write them using division (÷) and equals (=) signs
- Show that division of one number from another cannot be done in any order
- Solve problems involving division, using materials, arrays, mental methods and division facts, including problems in contexts

Key vocabulary:

Halve

Share, share equally

One each, two each, three each

Groups in pairs, threes, tens

Equal groups of

÷, divide, divided by, divided into

Left, left over

Strategies:

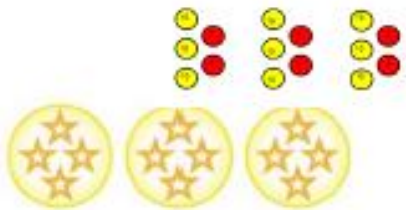
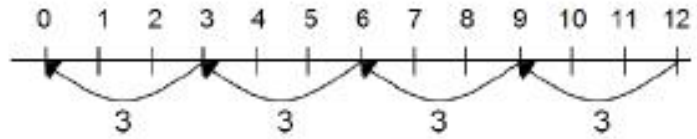
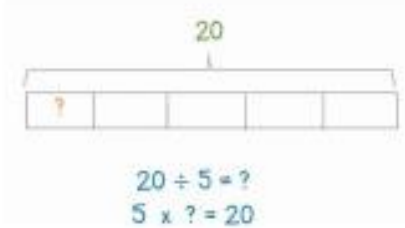
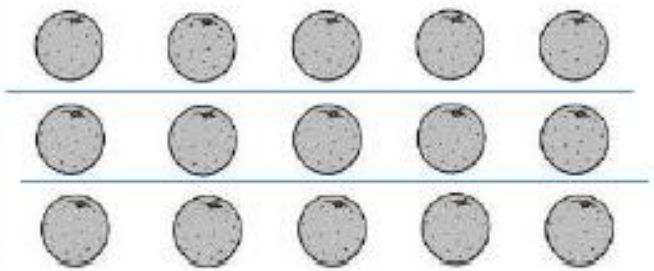
- Division as grouping
- Division within arrays

Mental Strategies:

- Children should experience regular counting on and back from different numbers in 1s and multiples of 2, 3, 5 and 10
- Children who are able to count in twos, threes, fives and tens can use this knowledge to work out other facts such as 2 x 6, 5 x 4, 10 x 9
- Children should continue to develop understanding of division as both sharing and grouping

$15 \div 3 = 5$

15 pencils shared between 3 pots, how many in each pot?

Objective and Strategies	Concrete	Pictorial	Abstract
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.   96 ÷ 3 = 32 	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.  	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?
Division within arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	 Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7

Fractions

Recognise, find, name and write fractions bI, $\frac{1}{4}$, $\frac{3}{4}$, 2/4 of a length, shape, set of objects or quantity.

Write simple fractions for example, $\frac{1}{2}$ of 6 = 3 and recognise the equivalence of $\frac{1}{2}$ and 2/4.

Year 3
Division

Pupils should be taught to:

- Recall and use division facts for the 3, 4 and 8 multiplication tables
- Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers divided by one-digit numbers, using mental and progressing to formal written methods
- Solve problems, including missing number problems, involving division, and correspondence problems in which n objects are connected to m objects

Key vocabulary:

Halve

Share, share equally

One each, two each, three each

Groups in pairs, threes, tens

Equal groups of

÷, divide, division, divided by, divided into

Left, left over, remainder

Strategies:

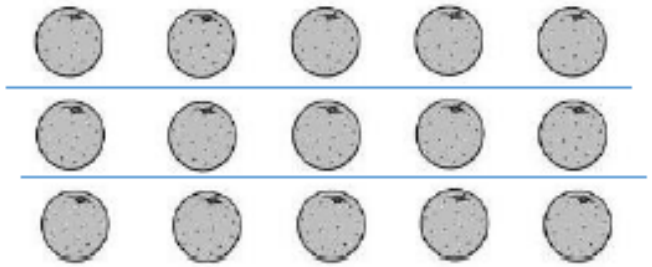
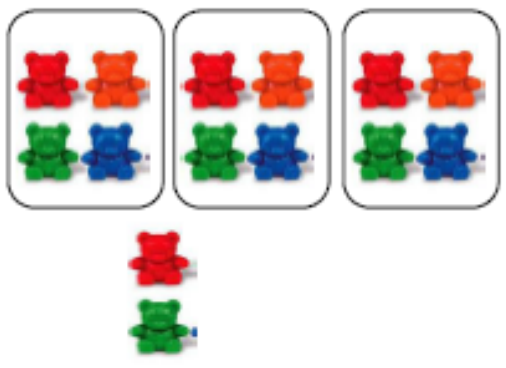
- Division within arrays
- Division with a remainder
- Short division (2 digits by 1 digit – concrete and pictorial)

Mental Strategies:

- Children count regularly, on and back, in steps of 3, 4 and 8
- Children are encouraged to use what they know about times table facts to work out other times tables – this then helps them to make new connections
- Children will make use of multiplication and division facts they know to make links with other facts:

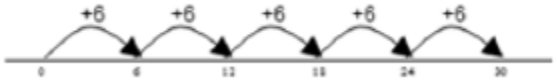
  $3 \times 2 = 6, 6 \div 3 = 2, 2 = 6 \div 3$
  $30 \times 2 = 60, 60 \div 3 = 20, 2 = 60 \div 30$

- They should be given opportunities to solve grouping and sharing problems practically
- Children should be given the opportunity to further develop understanding of division (sharing) to be used to find a fraction of a quantity or measure

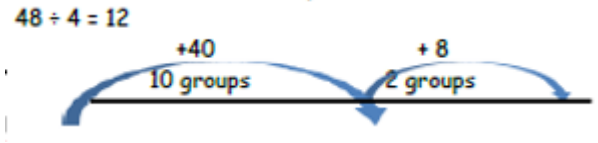
Objective and Strategies	Concrete	Pictorial	Abstract
Division within arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	 Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7
Division with a remainder	14 ÷ 3 = Divide objects between groups and see how much is left over 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder. 	Complete written divisions and show the remainder using r. 

Written Methods:

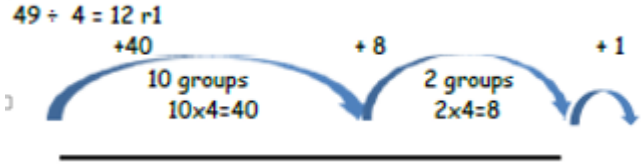
- Continue using a range of equations as in year 2 but with appropriate numbers
- Grouping: how many 6's are in 30?



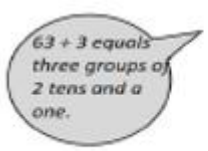
- Becoming more efficient using a number line: children need to be able to partition the dividend in different ways

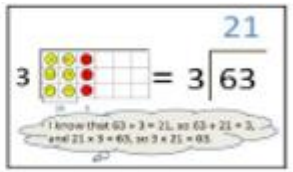


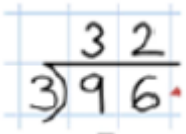
- Remainders:



- Place value counters can be used to support children apply their knowledge of grouping. Once children are secure with division as grouping and demonstrate using number lines/ arrays, short division for larger 2-digit numbers should be introduced, initially with carefully selected examples requiring no calculating of remainders at all. Start by comparing it to an array.







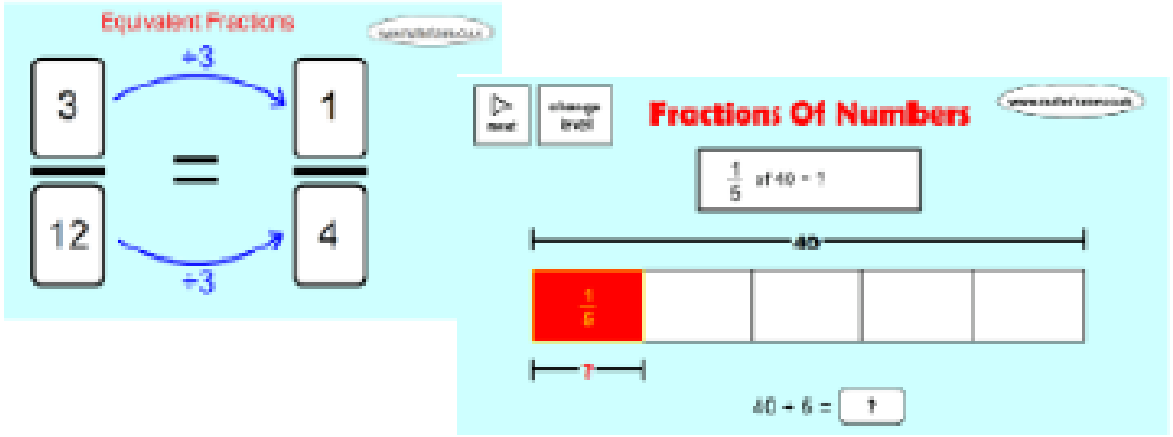
Fractions

Use children's intuition to support understanding of fractions as an answer to a sharing problem.

3 apples shared between 4 people = $\frac{3}{4}$

Recognise and show, using diagrams, equivalent fractions with small denominators.

Recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators.



Year 4
Division

Pupils should be taught to:

- Use division facts for multiplication tables up to 12 x 12
- Use place value, known and derived facts to divide mentally, including dividing by 1

Key vocabulary:

Halve

Share, share equally

One each, two each, three each

Groups in pairs, threes, tens

Equal groups of

÷, divide, division, divided by, divided into

Left, left over, remainder

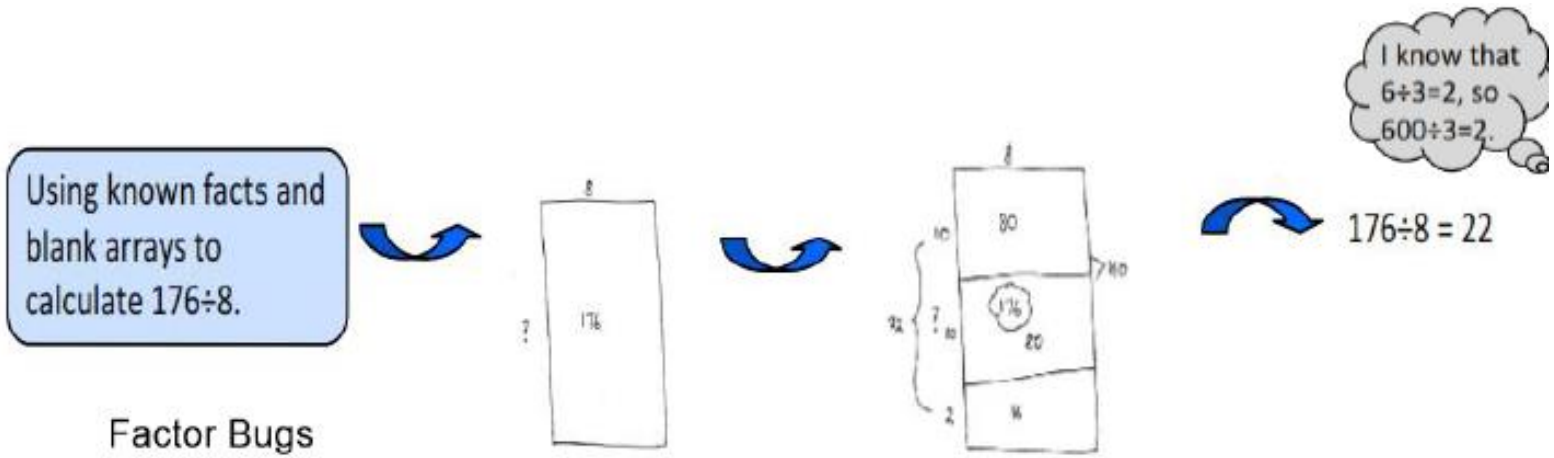
Factor, quotient, divisible by, inverse

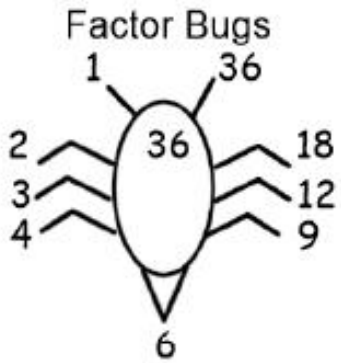
Strategies:

- Division within arrays
- Division with a remainder
- Short division (up to 3 digits by 1 digit – concrete and pictorial)

Mental Strategies:

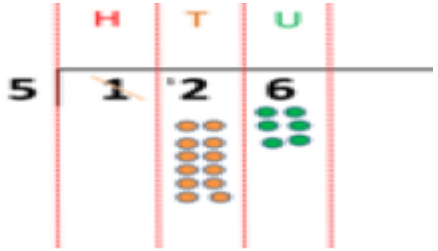
- Recognise and use factor pairs and commutativity in mental calculations



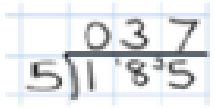


Written Methods:

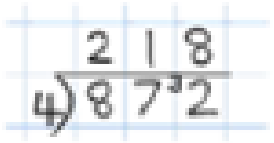
- Short division to be modelled for understanding using place value counters as show below. Calculations with 2 and 3-digit dividends.



- Pupils move on to dividing numbers with up to 3-digits by a single digit, however problems and calculations provided +should not result in a final answer with remainder at at this stage.



- When the answer for the first column is zero, children should initially write a zero above to acknowledge its place.



Year 5
Division

Pupils should be taught to:

- Divide numbers mentally drawing upon know facts
- Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- Divide whole numbers and those involving decimals by 10, 100 and 1000

Key vocabulary:

Halve

Share, share equally

One each, two each, three each

Groups in pairs, threes, tens

Equal groups of

÷, divide, division, divided by, divided into

Left, left over, remainder

Factor, quotient, divisible by, inverse

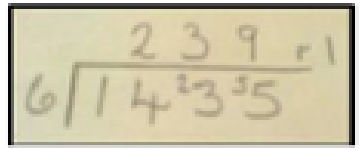
Strategies:

- Short division (up to 4 digits by a 1 digit number, interpret remainders appropriately for the context)

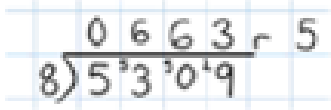
Mental Strategies:

- Identify multiples and factors, including all factor pairs of a number, and common factors of two numbers
- Pupils apply all the multiplication tables and related division facts frequently and use them confidently

Written Methods:



- The language of grouping to be used (see year four)
- Children begin to practically develop their understanding of how to expres the remainder as a decimal or a fraction. Ensure practical understanding allows children to work through this.



- Short division with remainders: division now has to have a real life problem solving context, where pupils consider the meaning of the remainder and how to express it (as a fraction, decimal or rounder number or value, depending on the context of the problem).

The answer to 5309 ÷ 8 could be expressed as 663 and five eighths, 663 r 5, as a decimal, or rounded as appropriate to the problem involved.

98 ÷ 7 becomes

$$7\overline{)98} = 14$$

Answer: 14

432 ÷ 5 becomes

$$5\overline{)432} = 86 \text{ r } 2$$

Answer: 86 remainder 2

496 ÷ 11 becomes

$$11\overline{)496} = 45\tfrac{1}{11}$$

Answer: $45\frac{1}{11}$

$$5\overline{)432.0} = 86.4$$

Fractions

Recognise mixed numbers and improper fractions and convert from one form to the other.

Write mathematical statements > 1 as a mixed number.

Pupils connect equivalent fractions > 1 that simplify to integers with division and other fractions > 1 to division with remainders.

Pupils connect multiplication by a fraction to using fractions as operators (fractions of)

Link to division.

Pupils should make connections between percentages, fractions and decimals

Find me ¼ of 16. Find me ¾ of 16.

Year 6
Division

Pupils should be taught to:

- Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context.
- Perform mental calculations, including with mixed operations and large numbers
- Use their knowledge of the order of operations to carry out calculations involving the four operations
- **Key vocabulary:**

Halve

Share, share equally

One each, two each, three each

Groups in pairs, threes, tens

Equal groups of

÷, divide, division, divided by, divided into

Left, left over, remainder

Factor, quotient, divisible by, inverse

Strategies:

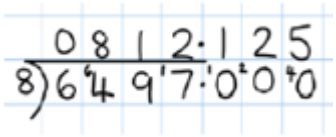
- Short division
- Long division (up to 4 digits by a 2 digit number – interpret remainders as whole numbers, fractions or round)

Mental Strategies:

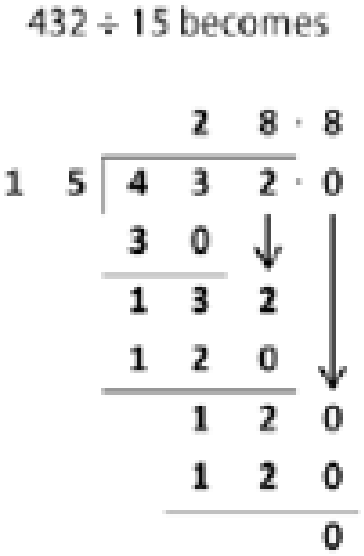
- Identify common factors
- Identify common multiples
- Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy
- Children should count regularly, building on previous work in previous years
- Children should practise and apply the multiplication facts to 12 x 12
- Pupils should be practising how to express remainders as fractions, decimals or use rounding, depending on the problem

Written Methods:

- Short division for dividing by a single digit



- Short division with remainders: pupils should continue to use this method, but with numbers to at least 4 digits, and understand how to express remainders as fractions, decimals, whole number remainders or rounded numbers. Real life problem solving contexts need to be the starting point, where pupils have to consider the most appropriate way to express the remainder.



Fractions

- Divide proper fractions by whole numbers (e.g. ⅓ ÷ 2 = ⅙)
- associate a fraction with division and calculate decimal fraction equivalents (e.g. 0.375) and for a simple fraction (e.g. ⅜)
- identify the value of each digit in numbers given to 3 decimal places and multiply and divide numbers by 10, 100 and 1000, giving answers up to 3 decimal places.
- use written division methods in cases where the answer has up to two decimal places
- recall and use equivalences between simple fractions, decimals and percentages, including in different contexts